EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS, VOL. 25, 837–855 (1996)

NUMBER OF MODES FOR THE SEISMIC DESIGN OF BUILDINGS

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SUMMARY

The effect of higher modes on the maximum response of buildings subjected to one horizontal component of earthquake ground motion is discussed with the objective of developing better design formulas for use in building design. Ideal buildings of different numbers of storeys and structural systems are defined ; their dynamic properties that define higher mode contribution are identified and are shown to be representative of real buildings. Design formulas that give the required number of modes to be used in a dynamic analysis are developed from parametric studies as a function of the admissible error, the number of storeys and the relation between the fundamental period and the corner spectrum period. The recommendations are simple to use and more rational and accurate than the ones actually in use in most seismic design codes.

KEY WORDS: number of modes; building codes; higher modes; building design; dynamic analysis; modal superposition

1. INTRODUCTION

One of the main advantages of the modal superposition method of dynamic analysis for buildings is that it is not necessary to calculate all vibrational modes. In order to optimize the calculations, the initial selection of a reduced number of modes is strictly necessary. Although many seismic codes specify a minimum number of modes for the analysis, in some cases they may lead to large errors in some response values because most recommendations do not include all the parameters that influence higher mode demands.

Many papers have dealt with the accuracy of the modal superposition method and the importance of higher modes but only a few have addressed the issue of the required number of modes. One of the earliest studies in this area was done by Clough[1] who pointed out the importance of the second and third modes, particularly for the shear forces developed in the upper storeys. In a later study[2] he concluded that the higher mode contribution becomes more significant as the period of vibration increases, that only the first mode response needs to be included in a deflection analysis and that considering only the first mode is quite inadequate for shear forces. Jennings[3] pointed out the importance of higher modes at the top floor acceleration. Cruz and Chopra[4] investigated the response of moment resisting plane frames to an ensemble of earthquake ground motions. The response contributions of the various vibration modes and their relations with the building properties were identified. Although only five-storey buildings with uniform properties were considered in the analysis, the results are indicated to be representative of the earthquake response of frames of a varying number of storeys. The results of this work provided a basis for developing improved simplified analysis procedures for preliminary building design.[5] In the opinion of the authors the most

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CCC 0098-8847/96/080837-19

Received 22 November 1994

Revised 20 February 1996

complete discussion of this topic has been recently presented by Chopra [6] who described the importance of several factors that contributes to the higher mode issue.

This study investigates the effects of higher modes with the objective of determining the required number of modes to use in an elastic dynamic analysis of buildings subjected to one horizontal component of ground motion. Buildings with different numbers of stories and structural behaviours are considered. Seismic motion is defined by typical code spectral shapes, including the overall effect of different soil conditions and earthquake distances.

2. HIGHER MODE CONTRIBUTION TO THE SEISMIC RESPONSE OF BUILDINGS

The contribution of higher modes varies with the response value in consideration. It is well known that storey displacements require less modes than shear storey forces, and that higher modes contribute less to the base overturning moment than to the base shear. Local member force behaviour can be related to overall building forces. Column moments are more affected by higher modes than beam moments and column axial forces because the former are closely related to the storey shears whereas the latter are closely related to the storey overturning moments.[4]

We assume that buildings are represented by linear systems with discrete masses concentrated at each floor and subjected to one horizontal component of ground motion given in terms of the acceleration response spectrum S_a. The buildings are assumed to have a regular plan configuration so that torsional dynamic effects are ignored. In mode i, the maximum modal values of the shear force at the base (V_{0i}), and at the top storey N (V_{Ni}), can be conveniently written as follows:

$$V_{0i} = \beta_i M S_{ai} \tag{1}$$

$$V_{Ni} = \alpha_i m_N S_{ai} \tag{2}$$

where S_{ai} is the acceleration spectral value for natural period T_i of mode i, M is the total building mass and m_N is the mass at the top storey.

The parameters β_i and α_i are the participation factors of mode i for the shear force at the base and the shear force at the top storey, respectively. They are given by

$$\beta_i = \frac{1}{M}\frac{L_i^2}{M_i} \tag{3}$$

$$\alpha_i = \phi_{Ni}\frac{L_i}{M_i} \tag{4}$$

where L_i is the earthquake excitation factor, M_i is the generalized mass[7] associated with mode i, and ϕ_{Ni} is the modal value at the top storey N in mode i.

It must be pointed out that these participation factors are dimensionless and independent of the normalizing criteria used for mode shape calculations. For the simple one degree of freedom system (one-storey building), all the participation factors are equal to one; therefore, for multistorey buildings they include the effects of the number of floors in addition to the shape of the mode in consideration. The participation factor for the base shear (β_i) is the well-known effective mass of mode i divided by the total building mass; it is a positive number and the sum for all modes is equal to one. The participation factor for the top storey (shear or displacement), α_i, can adopt positive or negative values but it can be shown[8] that the sum for all modes is also equal to one. This result can be found if the unit vector is written as a linear combination of the products of the modal vectors and the L_i/M_i values. The participation factors defined herein are found to be identical to the modal contribution factors defined by Chopra.[6]

Next we define the overall seismic response behaviour of the building by the maximum response values of the base shear (V_0) and the top storey shear (V_N). Since the buildings considered in this study have well-separated frequencies, we can neglect modal correlation and calculate the maximum response values by the square root of the sum of the squares of the individual modal maxima. Although recognizing the

limitations of this procedure to calculate maximum response values,[4] for the purpose of this study these estimations are considered to be adequate. If the resulting maximum response values are divided by the first mode response, we obtain the following response ratios that we define herein as the higher mode contribution:

$$\frac{V_0}{V_{01}} = \left[1 + \sum_{i=2}^{N} \left(\frac{\beta_i}{\beta_1}\frac{S_{ai}}{S_{a1}}\right)^2\right]^{1/2} \tag{5}$$

$$\frac{V_N}{V_{N1}} = \left[1 + \sum_{i=2}^{N} \left(\frac{\alpha_i}{\alpha_1}\frac{S_{ai}}{S_{a1}}\right)^2\right]^{1/2} \tag{6}$$

Therefore, the higher mode contribution depends on the ratios of the modal participation factors (α_i/α_1 and β_i/β_1) and on the ratios of the modal spectrum accelerations. Now, the ratios of the participation factors depend on the type of structural system and the number of storeys as will be shown later. The ratios of the modal spectrum accelerations depend on the spectrum shape, the spacing of the system natural frequencies which can also be described in terms of the natural frequency ratios ω_i/ω_1 and, primarily, on the location of the natural frequencies on the response spectrum.

From formulas (3) and (4) we can write the following relation:

$$\frac{\beta_i}{\beta_1} = \frac{\alpha_i}{\alpha_1}\left(\frac{L_i}{L_1}\frac{\phi_{N1}}{\phi_{Ni}}\right) \tag{7}$$

Since this ratio does not depend on the mode normalizing criteria, we can assume a unit modal value at the top for all modes ($\phi_{Ni} = \phi_{N1} = 1$) and examine the trends of equation (7) for i greater than 1. Due to the change in signs that show the modal co-ordinates for higher modes, L_i should be less than L_1 and we can say that the β_i/β_1 ratios should be smaller than the α_i/α_1ratios for most modes. Numerical results of these ratios shown later in Tables I and II for the ideal buildings support this finding. Therefore, from equations (5) and (6) we then concluded that the top storey shear has more contribution of higher modes than the base shear. Furthermore, from additional numerical results,[9] it is concluded that the top storey shear has more contribution of higher modes than a shear force at any other storey. This conclusion is also supported by numerical results presented in Reference 6 for five-storey buildings with several beam-to-column stiffness ratios. Therefore we can conclude that the top storey shear is the response parameter that has the largest contribution of higher modes. It should be remarked that local member forces associated to top storey shear would exhibit a similar contribution.

3. BUILDINGS AND GROUND RESPONSE SPECTRA

3.1. Ideal buildings

Ideal buildings are represented in this study by plane structures with equal mass at each floor and constant storey height. Three ideal buildings with different structural systems are defined as follows: (1) a Shear Building with Constant Column Stiffness (SB-CCS), (2) a Shear Building with Variable Column Stiffness (SB-VCS) such that the first mode shape is linear[10] and (3) a Flexural Building (FB).

The shear buildings have beams of infinite stiffness, columns with only flexural deformation and may have any number of bays. At each floor level all columns are identical. The flexural building is a single column with constant stiffness. Axial deformations are neglected. The mode shapes of these ideal buildings are independent of the number of bays. Tables I–III show the dynamic parameters (β_i/β_1, α_i/α_1 amd ω_i/ω_1) that influence the higher mode contribution, as defined in Section 2, for the first five vibration modes and for 3, 8, 14, 20, 30 and 50 number of storeys. It must be pointed out that all these system parameters are independent of the value of the fundamental period.[8] Also, it is interesting to observe that frequency ratios are independent of the number of stories for the shear building with a linear first mode shape (SB-VCS) . As expected, for large values of N the frequency ratios for the shear building SB-CCS tend to the well-known values (3, 5, 7, 9, . . .) of the continuous uniform shear beam, whereas for the flexural building they tend to the

Table I. Modal participation factors for base shear

Ideal building	Number of storeys	β_1	β_2/β_1	β_3/β_1	β_4/β_1	β_5/β_1
	3	0·914	0·082	0·012	—	—
	8	0·856	0·107	0·035	0·015	0·007
Shear buildings	14	0·838	0·109	0·038	0·018	0·010
SB-CCS	20	0·830	0·110	0·039	0·019	0·011
	30	0·823	0·111	0·040	0·020	0·012
	50	0·818	0·112	0·040	0·020	0·012
	3	0·857	0·130	0·037	—	—
	8	0·794	0·143	0·054	0·028	0·016
Shear building	14	0·775	0·146	0·056	0·030	0·018
SB-VCS	20	0·767	0·146	0·057	0·030	0·018
	30	0·762	0·146	0·057	0·030	0·019
	50	0·757	0·146	0·057	0·030	0·019
	3	0·742	0·296	0·051	—	—
	8	0·656	0·313	0·109	0·053	0·028
Flexural	14	0·636	0·310	0·108	0·055	0·022
building	20	0·629	0·308	0·107	0·055	0·033
FB	30	0·623	0·307	0·106	0·054	0·033
	50	0·619	0·307	0·106	0·054	0·033

Table II. Modal participation factors for top storey shear

Ideal building	Number of storeys	α_1	α_2/α_1	α_3/α_1	α_4/α_1	α_5/α_1
	3	1·220	− 0·230	0·049	—	—
	8	1·265	− 0·315	0·167	− 0·098	0·057
Shear buildings	14	1·270	− 0·327	0·188	− 0·127	0·090
SB-CCS	20	1·272	− 0·330	0·194	− 0·135	0·100
	30	1·273	− 0·332	0·197	− 0·139	0·106
	50	1·275	− 0·334	0·199	− 0·141	0·109
	3	1·286	− 0·259	0·037	—	—
	8	1·412	− 0·430	0·193	− 0·072	0·020
Shear building	14	1·449	− 0·490	0·280	− 0·150	0·072
SB-VCS	20	1·464	− 0·516	0·324	− 0·200	0·116
	30	1·476	− 0·537	0·363	− 0·250	0·167
	50	1·486	− 0·555	0·399	− 0·298	0·223
	3	1·294	− 0·283	0·055	—	—
	8	1·445	− 0·443	0·208	− 0·113	0·062
Flexural	14	1·493	− 0·489	0·254	− 0·160	0·108
building	20	1·514	− 0·508	0·274	− 0·180	0·128
FB	30	1·531	− 0·523	0·281	− 0·196	0·144
	50	1·545	− 0·535	0·304	− 0·210	0·1[illegible]3

values of the continuous uniform bending beam with constant mass and inertia.[12] Finally, because mode shapes are independent of the number of bays, therefore all system parameters given in Tables I–III are also independent of the number of bays.

3.2. Evaluation of ideal buildings

From the comparison of system parameters (Tables I–III) with the parameters calculated for real structures, it has been shown that the ideal buildings defined previously are adequate to represent the

Table III. Frequency ratios

Ideal building	Number of storeys	ω_2/ω_1	ω_3/ω_1	ω_4/ω_1	ω_5/ω_1
	3	2·80	4·05	—	—
	8	2·97	4·84	6·54	8·03
Shear building	14	2·99	4·94	6·84	8·65
SB-CCS	20	2·99	4·97	6·92	8·83
	30	3·00	4·99	6·97	8·93
	50	3·00	5·00	7·00	8·99
	3	2·45	3·87	—	—
	8	2·45	3·87	5·29	6·72
Shear building	14	2·45	3·87	5·29	6·72
SB-VCS	20	2·45	3·87	5·29	6·72
	30	2·45	3·87	5·29	6·72
	50	2·45	3·87	5·29	6·72
	3	5·28	10·9	—	—
	8	6·08	16·1	29·1	43·4
Flexural	14	6·20	17·0	32·5	51·7
building	20	6·24	17·3	33·4	54·3
FB	30	6·27	17·5	34·4	56·9
	50	6·27	17·5	34·4	56·9

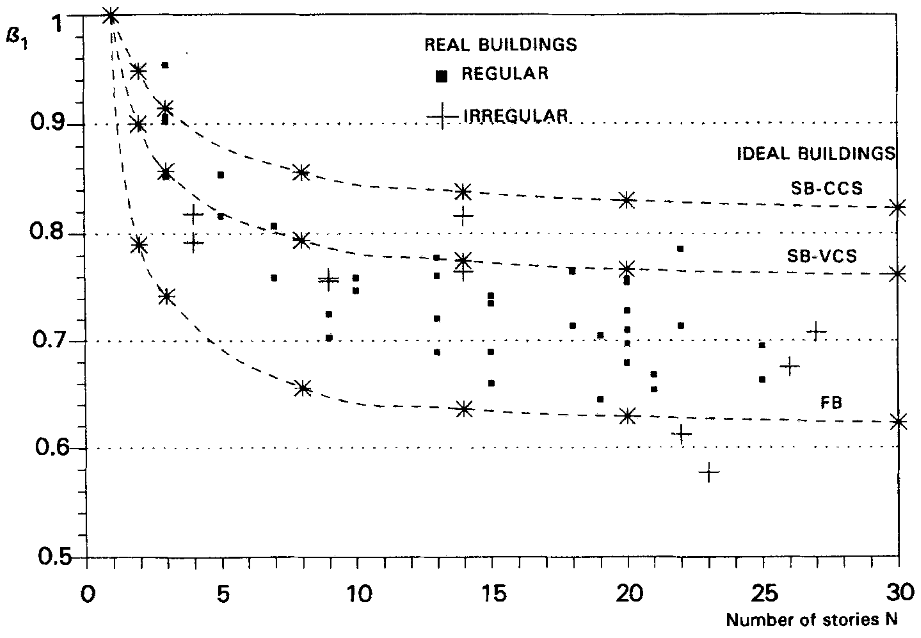

Figure 1. First mode participation factor (β_1) for base shear. Real buildings and ideal buildings

dynamic response behaviour of real buildings which are approximately regular in plan and which do not have large irregularities in its vertical distribution of mass and stiffnes.[9,11] Selected results are shown in Figures 1–3 for a group of reinforced concrete real buildings with system parameters calculated from refined mathematical models including flexural, shear and axial deformations in the elements (frames and walls) that form the lateral resisting system. These buildings were actually designed and built in the city of Caracas,

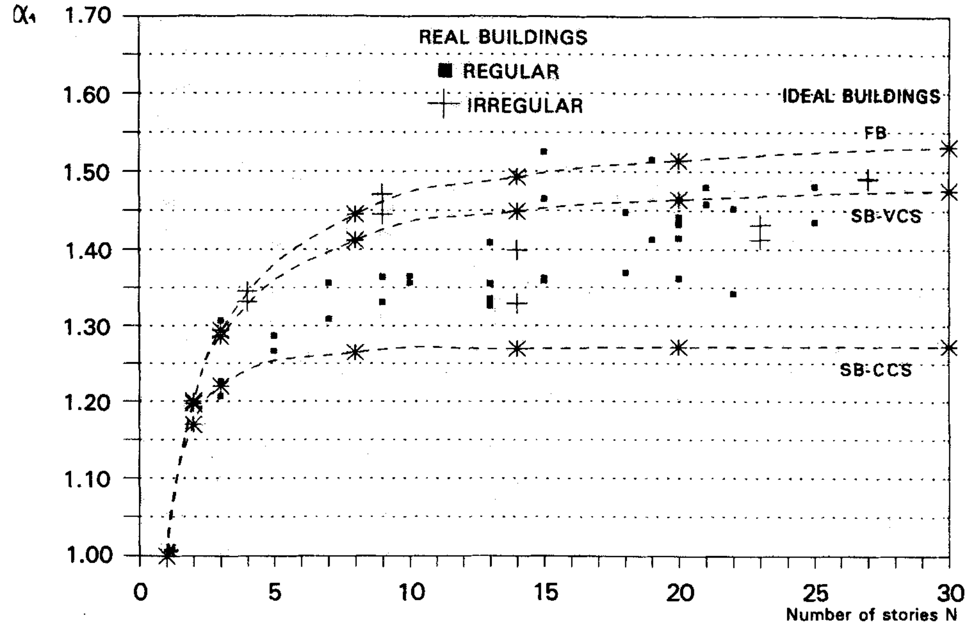

Figure 2. First mode participation factor (α_1) for top storey shear. Real buildings and ideal buildings

using design shear forces with an intensity and distribution quite similar to California requirements.[15] The first mode participation factors for the base shear (β_1) and for the top storey shear (α_1) are evaluated in Figures 1 and 2, respectively, for buildings of several number of storeys. Data points for the ideal buildings in Figures 1 and 2, and for all the buildings in Figure 3, are joined by linear segments in order to facilitate the interpretation of the results. It is remarkable that most data points for the real buildings fall inside the range defined by the ideal buildings.The ratios of the participation factors for the top storey shear (α_i / α_1) for the first five modes of the 25-storey ideal buildings are evaluated in Figure 3. From the 46 buildings analysed , 10 are defined as irregular because they have some type of mass or stiffness irregularity in its vertical configuration, according to the conditions specified in NEHRP.[13] In those buildings which have a significant reduction of dimensions and mass at the top storey, calculations of the α_1 values are made for the floor immediately below. From the examination of these results and others,[8,9,11] we can conclude that system parameters of real buildings are very well represented by parameters of the ideal ones. Real buildings with only sructural wall systems are better represented by the ideal flexural building (FB) whereas frame systems are better represented by the ideal shear buildings (SB-CCS or SB-VCS). Therefore, the ideal systems defined herein are suitable to investigate, by means of parametric studies, the effect of higher modes in the dynamic response of typical buildings.

3.3. *Ground response spectra*

Figure 4 shows the idealized smooth response spectrum used in this study. It is formed by a constant acceleration S_a (flat spectrum) for short periods followed by a constant velocity branch (hyperbolic acceleration spectrum) for larger periods, beginning at the corner period T^*. The general trends of this spectral shape are similar to the ones actually in use in several seismic codes. The corner period T^* will be used herein to introduce the effect of local soil conditions and earthquake distance on the spectral shape; as the soil conditions become softer or the site is subjected to larger-distance earthquakes, the corner period gets longer.

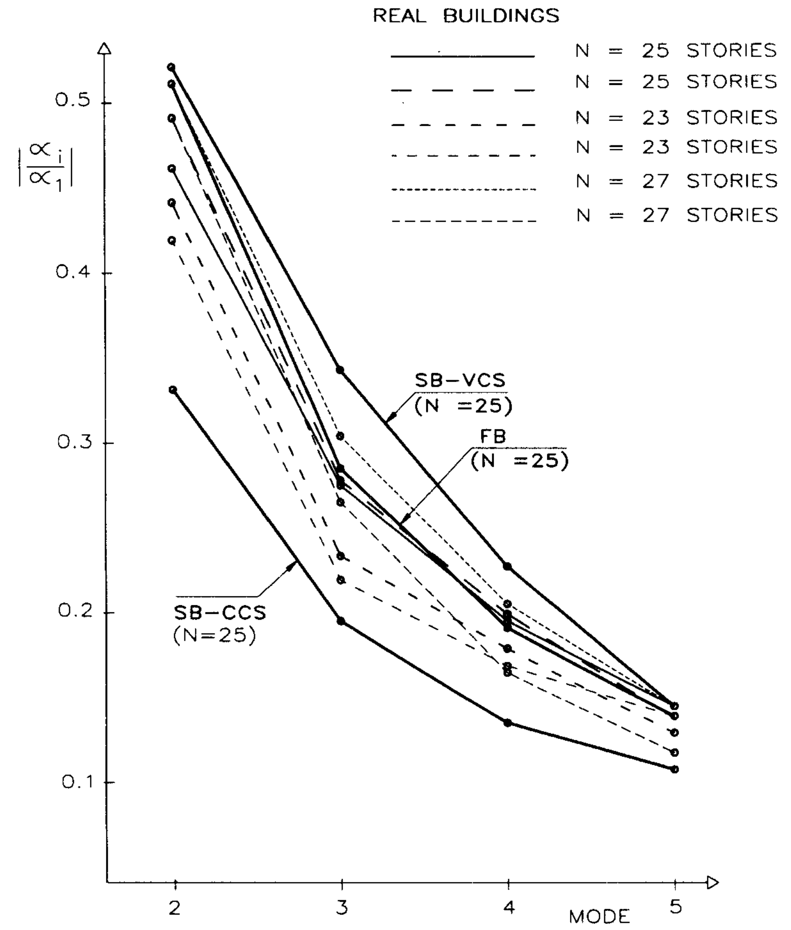

Figure 3. Participation factor ratios for top storey shear. Ideal and Real buildings

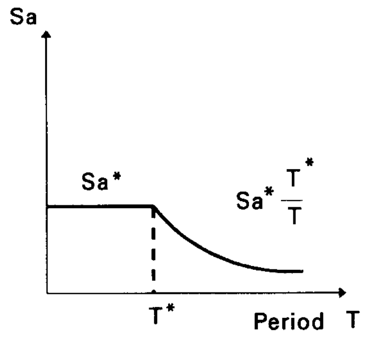

Figure 4. Response spectrum

4. HIGHER MODE CONTRIBUTION TO BASE SHEAR AND TOP STOREY SHEAR

Numerical results are presented next for the ideal buildings and the response spectrum defined in Figure 4. Figure 5 shows the higher mode contribution for the base shear (V_0/V_{01}) plotted against the ratio of fundamental period and corner spectrum period (T_1/T^*). Small values of T_1/T^* might be associated with

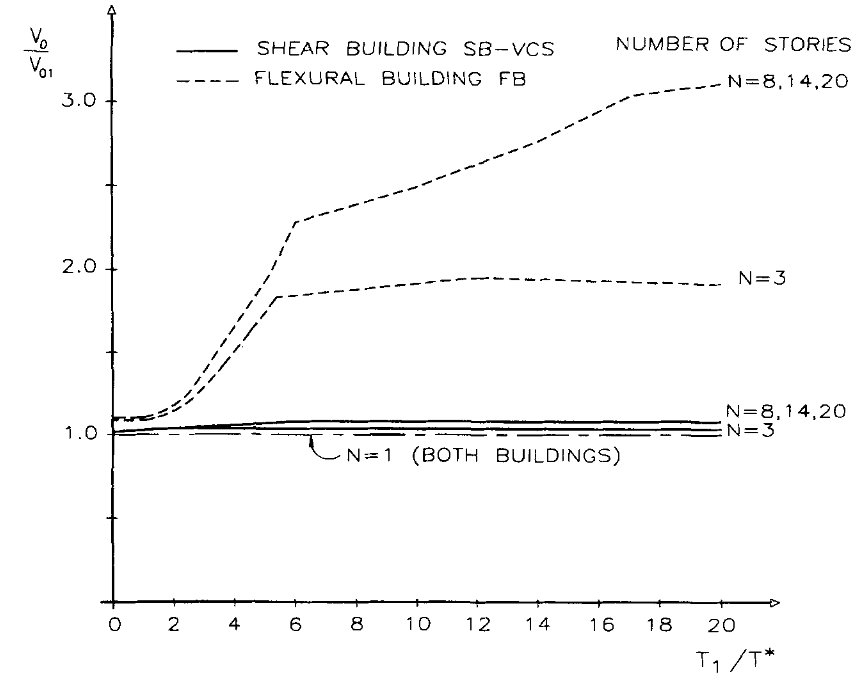

Figure 5. Higher mode contribution to base shear

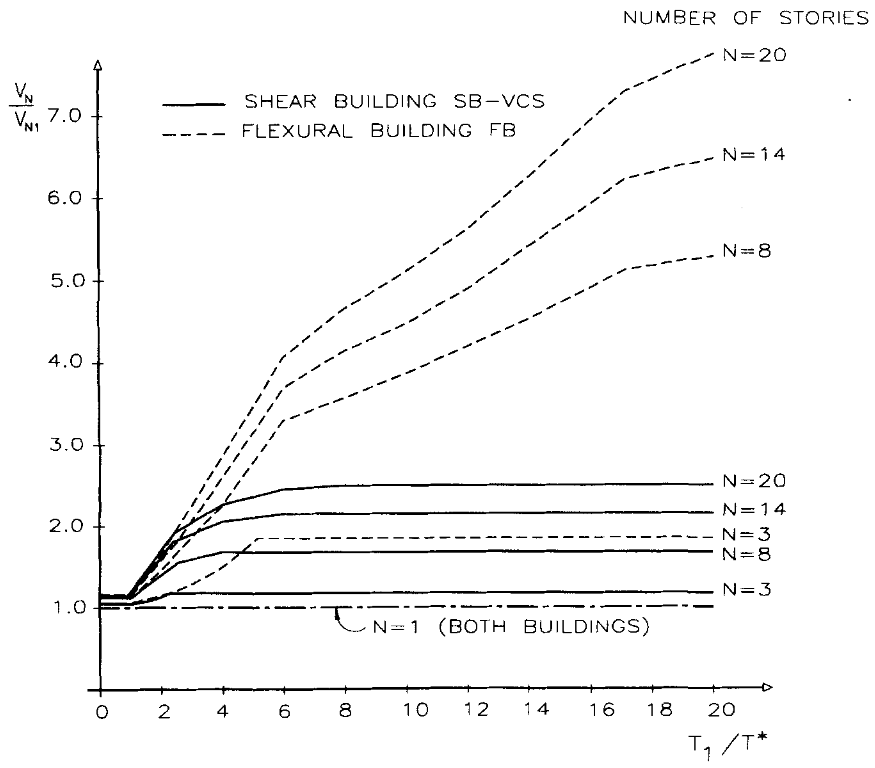

Figure 6. Higher mode contribution to top storey shear

short or tall buildings, located on a soft soil site, or short buildings located on stiff soils, whereas large values of T_1/T^* are associated with tall buildings on stiff soils. Results are shown for the FB and the SB-VCS buildings of 3, 8, 14 and 20 storeys. The wide range of plotted T_1/T^* values has been intentionally chosen in order to remark on the trends of the results . For buildings less than 20 storeys, the T_1/T^* values should not be greater than 7. A similar plot is presented in Figure 6 for the higher mode contribution to the top shear storey. The significance of parameter T_1/T^*, building type and number of storeys, is evident from these figures.

For large T_i/T^* values, the effects of higher modes can be much greater than the effect of the first mode. For example, for a 20-storey flexural building located on stiff soil (i.e. T_1/T^* is about 5), the higher modes contribute to the top story shear with a value which is about three times greater than the first mode contribution. Even for the base shear, the higher modes can double the contribution of the first mode for the flexural buildings in this range of large T_1/T^* values. Decreasing the T_1/T^* values leads to a significant reduction of the higher mode contribution; for example when T_1/T^* is less than one, the higher mode contributions only increase by 18 per cent for the first mode top storey shear and by 6 per cent for the first mode base shear.

Flexural buildings have a larger contribution of higher modes than shear buildings, particularly for the base shear, because the former have larger β_i/β_1 values (Table I) and also a larger spacing of natural frequencies (Table III). Although one class of shear buildings (SB-VCS) has α_i/α_1 ratios (Table II) similar to flexural buildings, the larger spacing of natural frequencies also leads to a larger contribution of higher modes at the top storey shear for the flexural buildings. Increasing the number (N) of storeys brings out a larger contribution of higher modes due mainly to the increase in participation factor ratios, especially for short buildings. This trend is particularly significant for the top storey shear and the flexural buildings.

5. ERRORS WHEN CONSIDERING A REDUCED NUMBER OF MODES

The relative errors which are introduced in the calculation when a reduced number of modes is considered in the analysis were determined as the difference between the exact value (calculated with all N modes) and the approximate value (calculated with a reduced number of modes), divided by the exact value.

The relative errors (e_0), in percentage, for the base shear of the 20-storey buildings are presented in Figure 7, plotted against the parameter T_1/T^*. The errors (e_N) for the top storey shear are shown in Figure 8. When only the first mode is included in the analysis, the larger errors correspond to the top storey shear of the flexural buildings, as expected; relative errors of 70 per cent are found when T_1/T^* is equal to 5. However, when the first two modes are included in the analysis of the top storey shear, larger errors are found for the shear buildings in most of the T_1/T^* range. The significance of the parameter T_1/T^* is evident from the results indicated in the figures. The errors for the base shear are always less than for the top storey shear.

It is interesting to compare the relative errors obtained for some real buildings with the errors that come from the ideal buildings. Table IV presents the errors calculated for the top storey shear of four reinforced concrete real buildings and the 25-storey ideal buildings, when 1, 2, 3, 4 or 5 modes are considered in the analysis. Results correspond to T_1/T^* values less than or equal to one. Real buildings 1 and 2 have 23 storeys with a structural system defined by a space frame; they have a large mass concentrated at the first floor which is about 16 per cent of the total mass of the building. Real buildings 3 and 4 have 27 storeys and a dual (shear wall–space frame) structural system with a somehow irregular vertical distribution of mass. The results confirm that the relative errors determined for the ideal buildings are in the range of the values obtained for real buildings, even if these have moderated irregularities in its vertical distribution of mass and stiffness.

6. NUMBER OF MODES FOR DESIGN AND COMPARISON WITH OTHER CRITERIA

6.1. *Number of modes required for an admissible error*

In order to derive design formulas for the required number of modes , we must adopt an admissible relative error, or tolerance, for the dynamic response values. Calculations were made for tolerances of 5 and 10 per cent. The results shown in Figure 9 correspond to a tolerance of 5 per cent. The ordinates in Figure 9(a) are

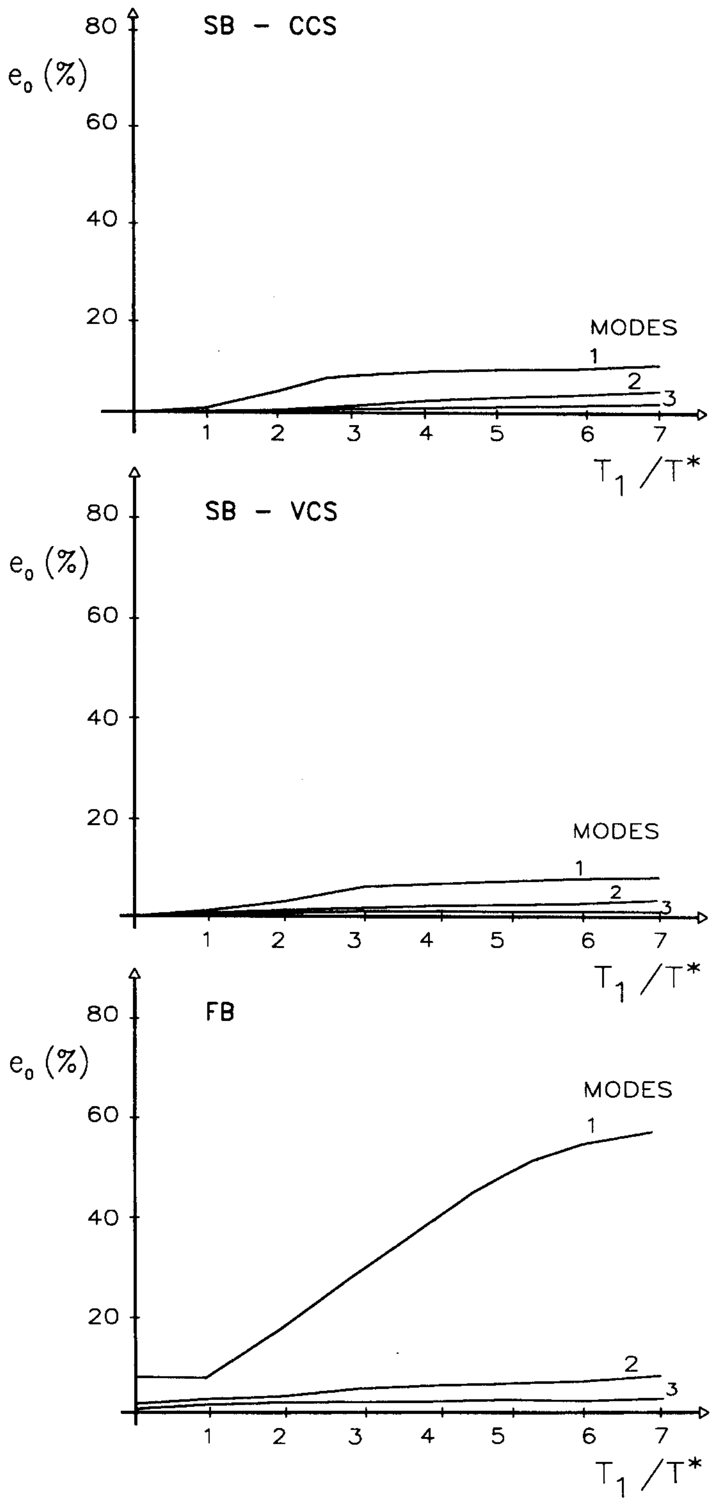

Figure 7. Base shear relative errors for 20-storey buildings

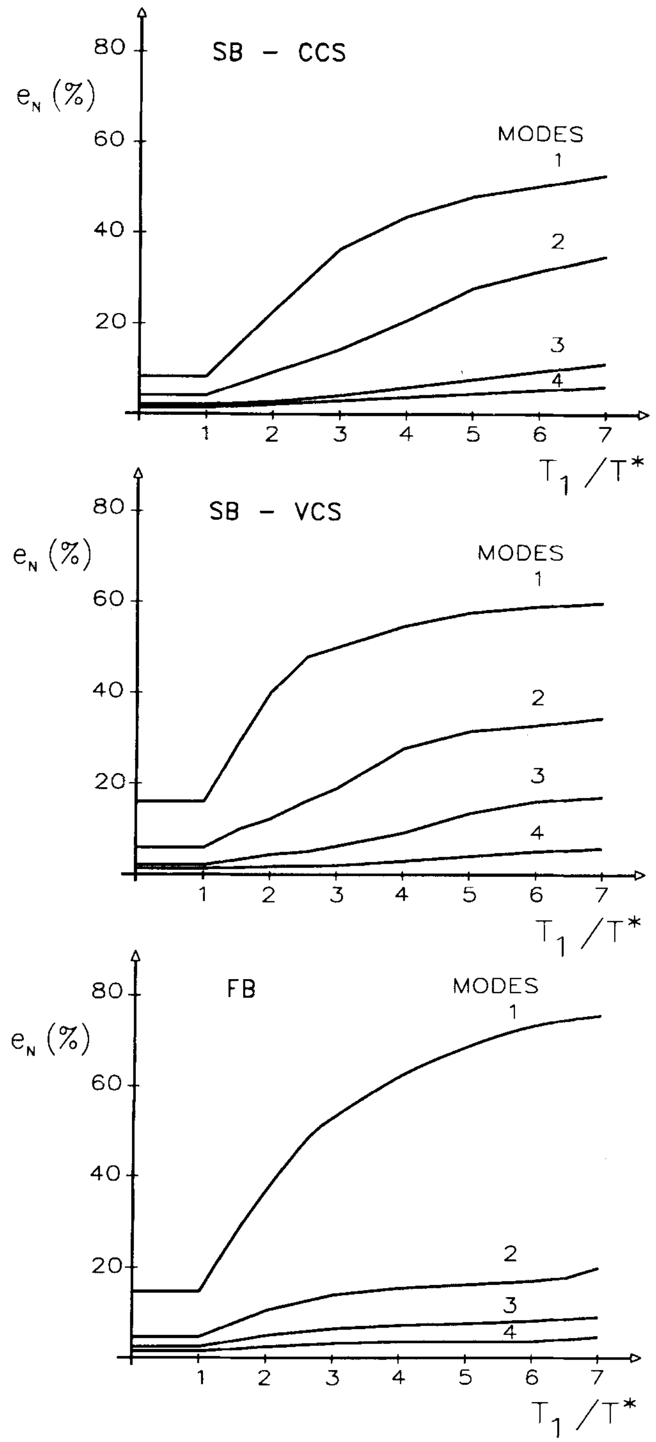

Figure 8. Top storey shear relative errors for 20-storey real and ideal buildings

Table IV. Top storey shear relative errors for 25-storey ideal buildings and four real buildings

Number of modes	Ideal buildings			Real buildings			
	SB-CCS	SB-VCS	FB	1	2	3	4
1	8·4	17·7	16·1	12·9	12·3	14·4	17·0
2	3·5	7·0	5·6	4·7	4·8	4·6	6·7
3	1·8	2·8	2·7	2·5	2·9	2·0	3·3
4	1·0	0·5	1·4	1·3	1·9	1·0	1·9
5	0·6	0·3	0·7	0·7	1·2	0·5	1·2

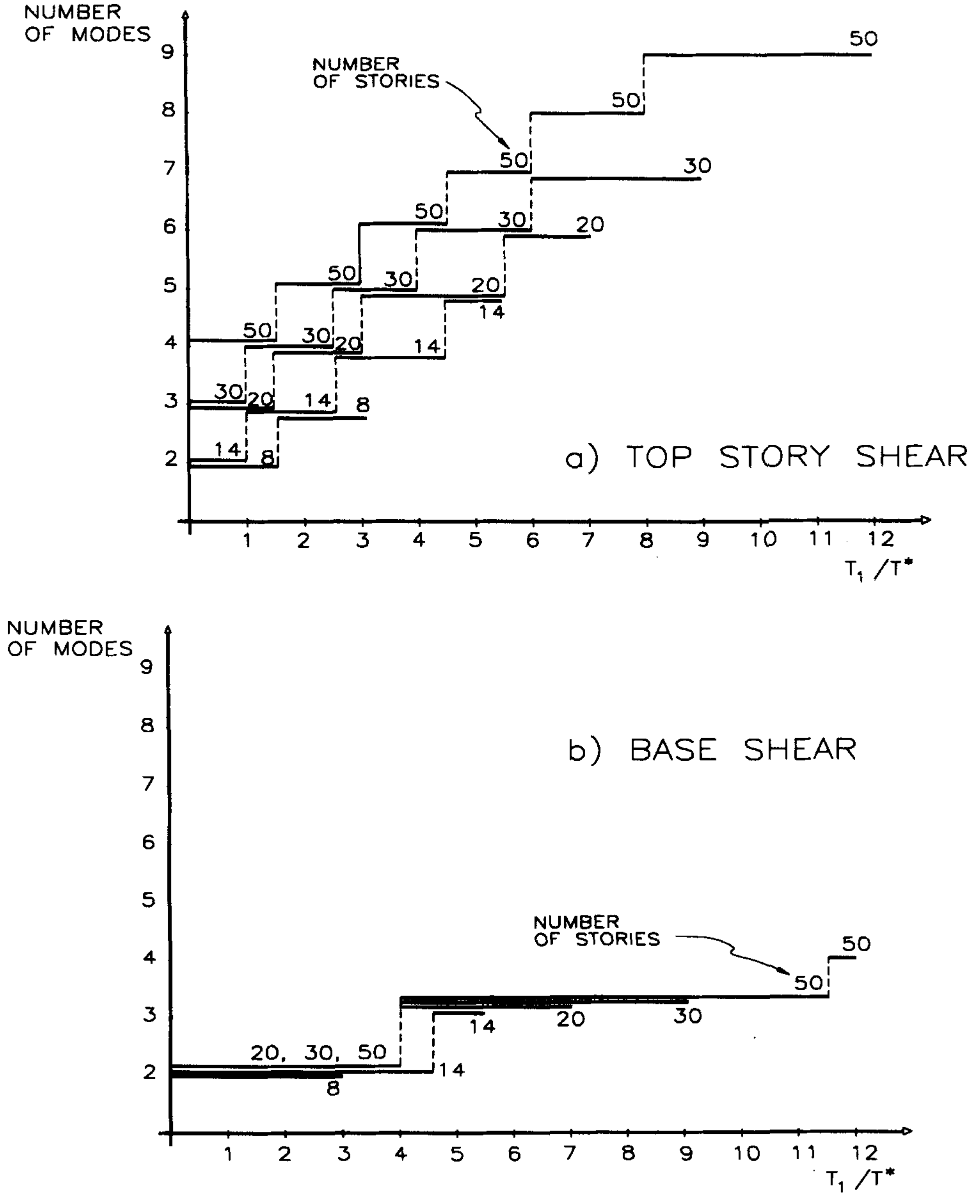

Figure 9. Number of modes required to keep relative errors below 5 per cent

the number of modes required to keep the relative error in the top storey shear below 5 per cent; the number of modes has been selected as the largest for the three ideal buildings. Therefore this is a somehow conservative value that is desirable in order to accommodate particular cases of real buildings with certain vertical irregularities. Results are plotted against the parameter T_1/T^*. The number of storeys (8–50) is indicated on each horizontal line of the figure. For each storey number the results are shown for a practical range of T_1/T^*values which correspond to limiting values for T_1 and T^*. Results for the base shear are presented in Figure 9(b). For example, and in reference to Figure 9(a), a 20-storey building with a flexural or shear structural system , located on a very soft soil (i.e. T_1/T^* is less than 1), only requires three modes in the dynamic analysis if we want to keep the error in the top storey shear below 5 per cent. If the same building is located on rock or very stiff soil (T_1/T^* is as greater as 7), then the required number of modes increases to 6. A 50-storey building on stiff soil requires as much as nine modes to keep errors below 5 per cent.

Simplified formulas that approximate the number (NM) of modes required to keep relative errors at any response parameter below a specified tolerance, were derived from results similar to the ones presented in Figure 9. To simplify the results, the number of storeys has been separated into two groups: one for buildings less than or equal to 20 storeys and the other for buildings greater than 20 storeys. The simplified formulas are:

(i) For relative errors less than 5 per cent

$N \leqslant 20$ storeys:

$$\mathrm{NM} = 3; \qquad T_1/T^* \leqslant 1{\cdot}5 \tag{8}$$

$$\mathrm{NM} = \tfrac{1}{2}(T_1/T^* - 1{\cdot}5) + 3; \qquad T_1/T^* > 1{\cdot}5 \tag{9}$$

$N > 20$ storeys:

$$\mathrm{NM} = 4; \qquad T_1/T^* \leqslant 1{\cdot}5 \tag{10}$$

$$\mathrm{NM} = \tfrac{2}{3}(T_1/T^* - 1{\cdot}5) + 4; \qquad 1{\cdot}5 < T_1/T^* \leqslant 6 \tag{11}$$

$$\mathrm{NM} = \tfrac{1}{2}(T_1/T^* - 6) + 7; \qquad T_1/T^* > 6 \tag{12}$$

(ii) For relative errors less than 10 per cent

$N \leqslant 20$ storeys:

$$\mathrm{NM} = 2; \qquad T_1/T^* \leqslant 1{\cdot}5 \tag{13}$$

$$\mathrm{NM} = \tfrac{1}{2}(T_1/T^* - 1{\cdot}5) + 2; \qquad T_1/T^* > 1{\cdot}5 \tag{14}$$

$N > 20$ storeys:

$$\mathrm{NM} = 3; \qquad T_1/T^* \leqslant 1{\cdot}5 \tag{15}$$

$$\mathrm{NM} = \tfrac{1}{2}(T_1/T^* - 1{\cdot}5) + 3; \qquad T_1/T^* > 1{\cdot}5 \tag{16}$$

The number that comes from these formulas must be approximated to the upper closest integer. For two-storey buildings, NM should be equal to two. These formulas are limited to buildings with the number of storeys not greater than 50 storeys.

The recommended formulas have been derived assuming the response spectrum shape shown in Figure 4 where the descending branch is proportional to $1/T$. The results obtained from these formulas are on the conservative side if we consider a descending branch proportional to $1/T^n$, for $n < 1$, due to the reduction of the spectrum acceleration ratios indicated in formulas (5) and (6).

6.2. Evaluation of formulas and comparison with other criteria

The recommended formulas to calculate the number of modes are next evaluated by means of dynamic analysis of real buildings and the results are compared with those obtained from the application of criteria specified by NEHRP,[13] UBC,[14] the Seismological Committee of the Structural Association of California SCSEAOC,[15] the Chile Seismic Code CHSC[16] and the Costa Rica Seismic Code CRS.[17] NEHRP requires

to include at least the lowest three modes of vibration or all modes with periods greater than 0·4 s. The UBC, SCSEAOC and CHSC require to consider all significant modes so that at least 90 per cent of the participating mass of the structure is included in the calculation of response. CRSC requires two modes for buildings less than eight storeys, plus one mode for each additional five storeys.

Two real buildings were considered as examples. Building 5 is an 18-storey reinforced concrete space frame building. Its structural plan is shown in Figure 10. The geometry and member sections of each frame in the Y direction are taken from Reference 18. The storey weight is 605 t, equal for all floors. Building 6 is also an 18-storey reinforced concrete building with a dual structural system (frames and walls) as shown in Figure 11. The column and beam dimensions are the same as in building 5. The structural walls are 20 cm thick. It has a constant storey weight equal to 1536 t. Tables V and VI show the dynamic properties (T_i, α_i, β_i) of the first vibration modes for buildings 5 and 6 respectively. Building 5 has a fundamental period of 2·1 s whereas building 6 has 1·49 s.

Both buildings were analysed by standard dynamic analysis computer programs including axial, shear and bending deformations and considering only one horizontal dynamic degree of freedom per storey in the Y direction. Two response spectral shapes were considered: spectrum 1 is the spectrum used in this study

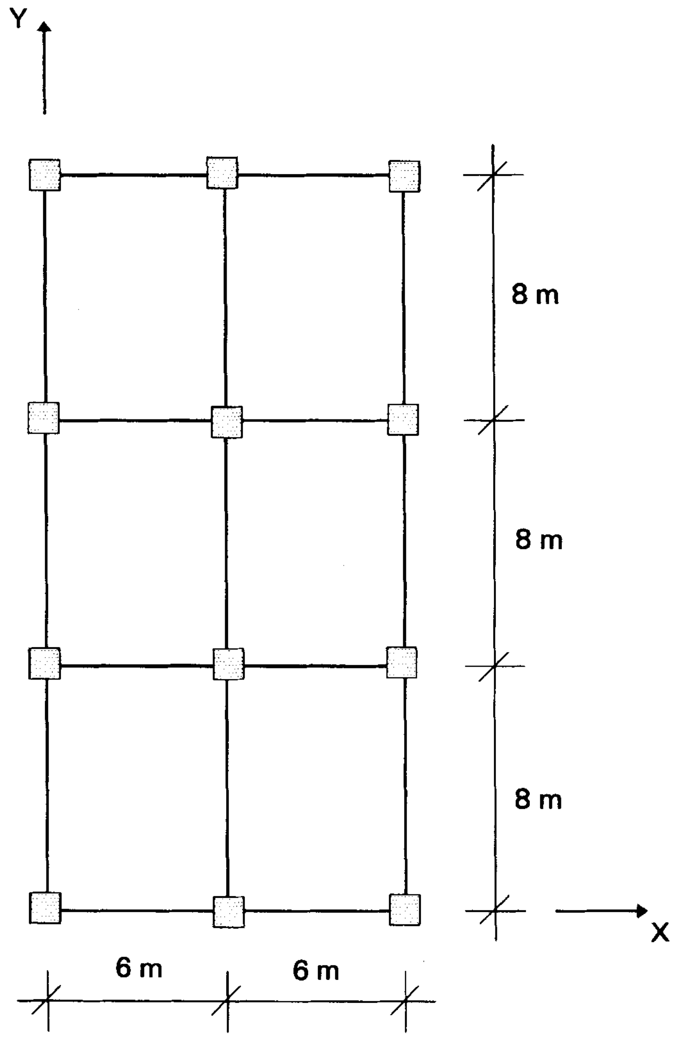

Figure 10. Building 5

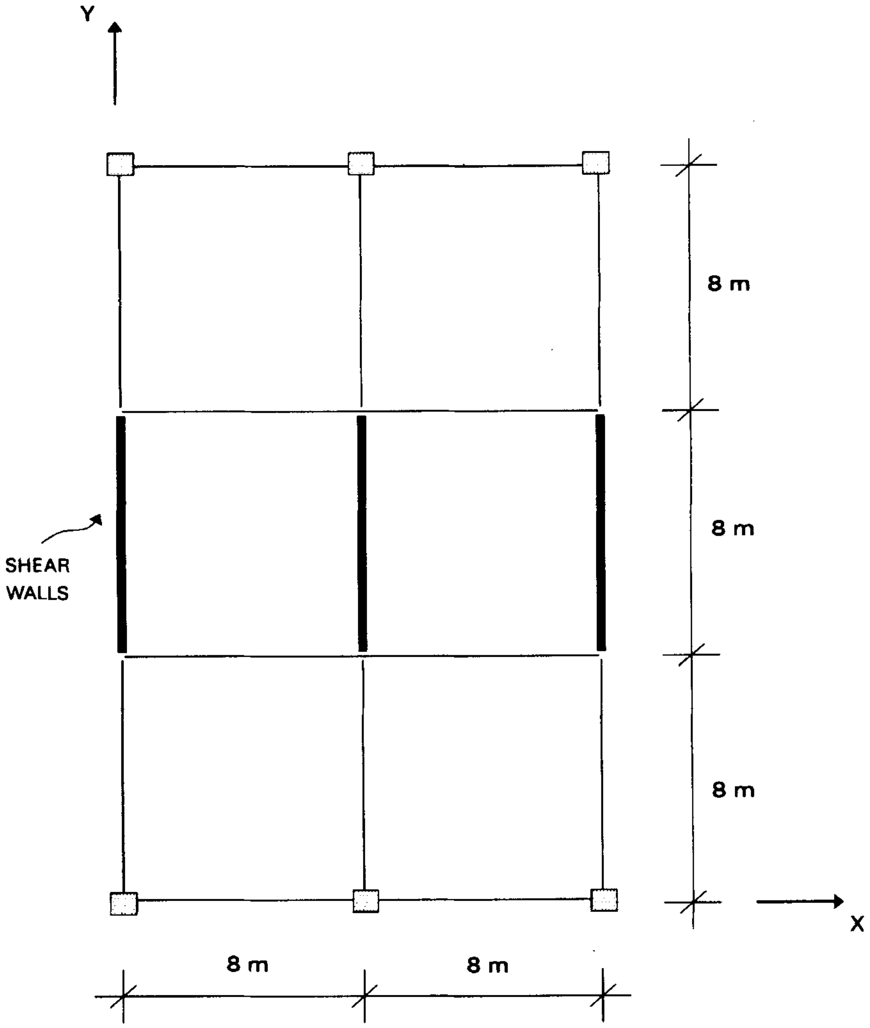

Figure 11. Building 6

Table V. Dynamic properties of building 5

Mode	T_i	α_i	β_i
1	2·10	1·40	0·719
2	0·77	− 0·62	0·137
3	0·45	0·36	0·051
4	0·31	− 0·25	0·025
5	0·23	0·18	0·016
6	0·18	− 0·14	0·012

Table VI. Dynamic properties of building 6

Mode	T_i	α_i	β_i
1	1·49	1·51	0·643
2	0·36	− 0·77	0·194
3	0·16	0·42	0·072
4	0·10	− 0·26	0·036
5	0·070	0·17	0·018
6	0·057	− 0·13	0·011

(Figure 4) which has a descending branch proportional to $1/T$; spectrum 2 has a descending branch proportional to $1/T^{2/3}$, which is used in NEHRP. For both spectra, the corner spectrum period is assumed to vary from 0·3 s (associated with rock or very stiff soils and close-distance earthquakes) to 1·5 s (associated with very soft soils and long-distance earthquakes). For each case, response values were calculated considering all of the building modes of vibration and also considering the reduced number of modes given by this study for a tolerance of 5 per cent formulas (8)–(12)) and the numbers of modes

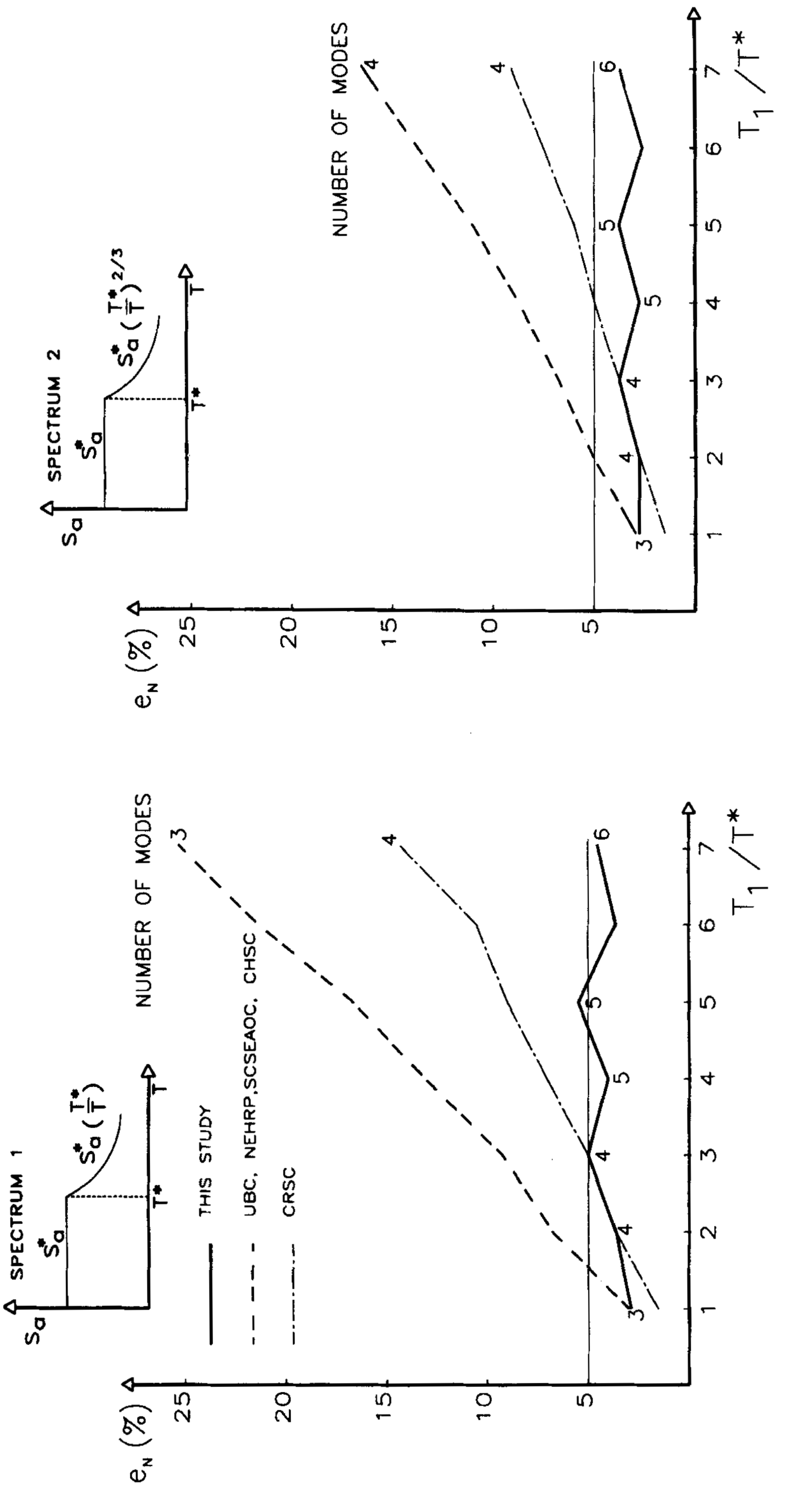

Figure 12. Top storey shear relative errors determined from this study and from some building codes, for a real 18-storey space frame building

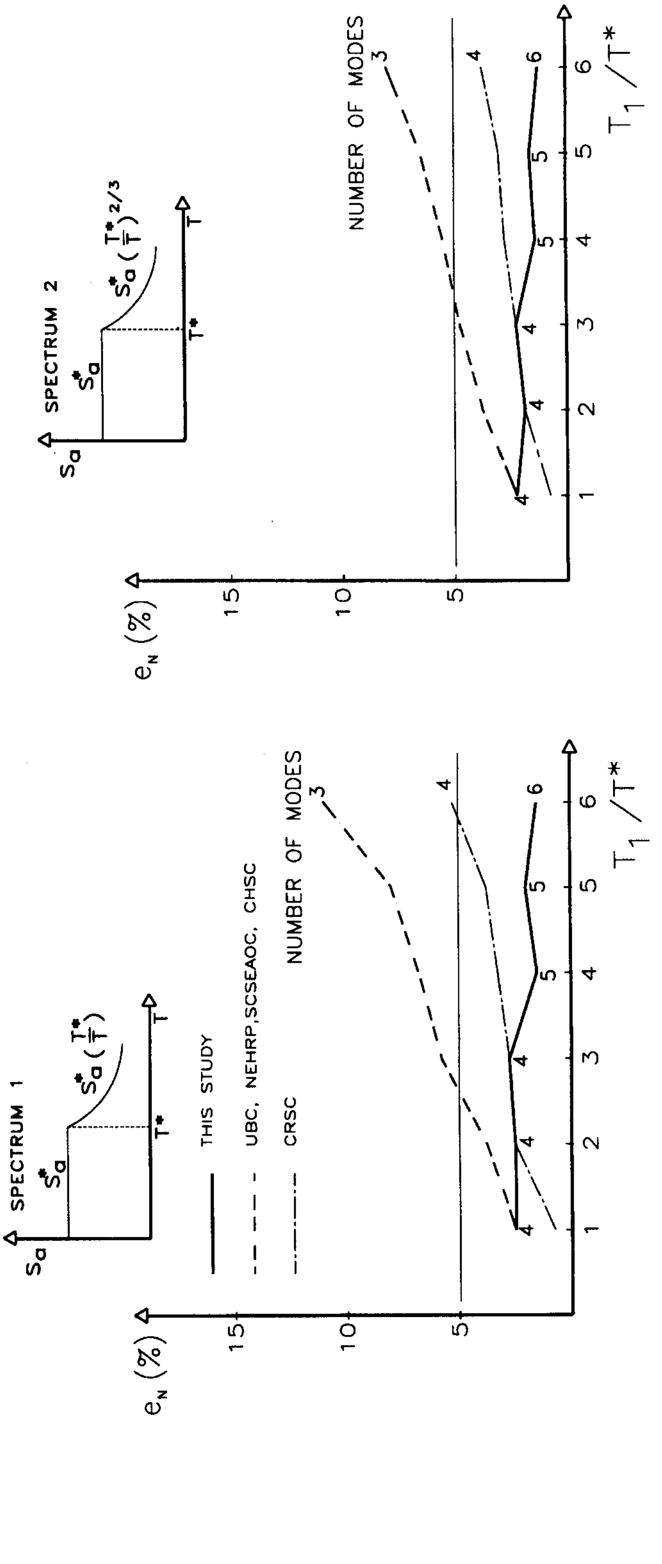

Figure 13. Top storey shear relative errors determined from this study and from some building codes for a real 18-storey dual building

recomended by NEHRP, UBC, SCSEAOC, CHSC and CRSC. Relative errors were calculated for several response parameters; the top storey shear was found to yield the largest relative error for all cases.

Results are presented in Figures 12 and 13 for buildings 5 and 6, respectively, and for spectra 1 and 2. The relative error (e_N) at the top storey shear is plotted against the parameter T_1/T^*. The number of modes required by each criterion is also indicated in the figures. An examination of both figures indicates that the number of modes given by this study leads to errors that are approximately constant for all T_1/T^* values. This means that errors are somehow independent of the soil conditions and/or of the earthquake distance. On the other hand, all the other criteria lead to errors that increase monotonically with T_1/T^* values, which means that the same building would be designed with more accuracy if it is located on a soft soil rather than on a stiff soil. The largest relative errors are observed for building 5 when the UBC and NEHRP criteria are applied. For this case both criteria allow to use three modes for the analysis. The maximum error is 16 per cent for spectrum 2 and 26 per cent for spectrum 1. On the other hand, the proposed formulas yield relative errors below the specified tolerance (5 per cent) for practically all cases.

7. CONCLUSIONS

(i) To investigate higher mode contribution, the three ideal buildings considered in this study are representative of real buildings which are approximately regular in plan and which have moderated irregularities in its vertical distribution of mass and stiffness. Dynamic parameters that influence the higher mode contribution are independent of fundamental period and the number of bays of the structural system. They do depend on the number of storeys.

(ii) The top storey shear is the overall response parameter that shows the largest contribution of higher modes. Local member forces associated to the top storey shear will exhibit a similar higher mode contribution. Higher mode contribution increases with increasing values of the parameter T_1/T^* and with increasing number of storeys. Also, flexural buildings have a larger contribution of higher modes than shear buildings. For tall flexural buildings resting on stiff soils (i.e. T_1/T^* about 5) and a spectrum with a descending branch proportional to $1/T$, the higher mode contribution to the top storey shear could be three times greater than the first mode contribution. For tall buildings located on stiff soils, the higher mode contribution to the top storey shear can double the contribution to the base shear.

(iii) Simplified formulas ((8)–(16)) that give the number of modes required to keep relative errors in response values below specified tolerances of 5 or 10 per cent were obtained. Half the number of modes required by the top storey shear are enough to keep the errors for the base shear below the specified tolerance. For a 5 per cent tolerance, any 20-storey building on a very soft soil would only require three modes, whereas if it is on a stiff soil the required number of modes may increase to six.

(iv) Comparison with other criteria specified by several building codes indicates that these formulas lead to dynamic response values that are more rational and accurate because they take into account the response spectrum in addition to the building dynamic properties. For tall buildings, dynamic response values calculated from UBC and NEHRP procedures lead to relative errors that increase monotonically with T_1/T^* values; i.e. the same building would be designed with more accuracy if it is on soft soil rather than stiff soil. For example, for a real 18-storey building (space frame) on stiff soil and a spectrum with a descending branch proportional to $1/T^{2/3}$, the relative errors are about 16 per cent for the top storey shear and associated column shear forces and moments. If the descending branch is proportional to $1/T$, the relative errors are as large as 26 per cent. On the other hand, for all cases the proposed formulas lead to relative errors below the specified 5 per cent tolerance.

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